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ABSTRACT

This document notes that arithmetic disabilities can occur with an otherwise normal intelligence level. Two models of explanation are given. The first emphasizes the conditions under which children learn mathematics, the second views the identification of a number of factors in the "structure of intelligence" that appear to be strongly related with success in the teaching of mathematics. The starting point for this research is the question whether mathematics ability in primary schools is related with any specific intellectual factors. Independent variables selected for study included intelligence, specific factors such as visual-spatial abilities, memory and fluency, and abilities in classification and seriation. One-hundred twenty pupils between the ages of 9 and 11 that were judged by their teachers to be one to two years behind in mathematics were selected for the study. The study concluded that, in general, arithmetic abilities are more dependent on verbal and non-verbal problem solving than on general mental ability. Addition and subtraction are distinguished from general mathematical ability as they appear to be dependent on different abilities. It is suggested that further research should aim at task-analysis of the different tasks. (MP)

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ARITHMETIC DISABILITIES

The relation between arithmetic and some psychological abilities.

A reanalysis

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Introduction

In the field of learning disabilities by far the most attention is paid to reading and writing disabilities, dyslexia and dysorthographia, while considerably less attention is paid to e.g. arithmetic disabilities.
It is not very clear why this difference in interest in these two kinds of learning problems should exist. Possibly the opinion is rather widespread that reading and writing disabilities depend more on specific factors, independent from intelligence, whereas arithmetic disabilities bring out the level of intelligence and for this reason they would be less capable of influence and improvement.
Since it has become clear, however, that arithmetic disabilities can also occur with an otherwise normal intelligence level, the question after the relation with other psychological factors, aspects or abilities has become typical. In theory two models of explanation are possible, setting aside the general intelligence level as a hypothesis accounting for arithmetic disabilities.
In the first place Schonell and Schonell (1965) have emphasized the conditions in which the child has to learn arithmetic. Too little practical experience, bad instruction, too lop-sided memorizing education, negative motivation, emotional consequences of psychical stress and anxiety to fail are unfavourable conditions to be successful in arithmetic, besides problems of concentration and hyperactivity.
In the second place in the psychology of the structure of intelligence the identification of a number of factors that appeared to be related with success in the teaching of mathematics is well-known. (Vernon, 1971; Borghouts, 1976; Smith, 1964; Butcher, 1970; Eysenck, 1967). Particularly the so-called visuo-spatial factor, indicated by the symbol k, or combined with the mechanical ability indicated as k : m, appeared to be the most important predictor together with the general intelligence factor g, as far as branches such as mathematic are concerned. Although this spatial ability seems to develop in the period between the eleventh and fifteenth year (Vernon, 1971; Fruchter, 1954), it is still a question whether arithmetic at the primary school is

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also related with certain intellectual factors maybe also with visual and spatial abilities in particular.
This question has been the starting-point of the here-mentioned research.

The problem set

If arithmetic is based on, or related to specifiec conditions or certain factors that can be pointed out in the structure of the intelligence, then the question is which are these factors.
An inquiry into these aptitudes can take place by posing the question for underlying constructs that could explain the connexion among varibales. Factor-analysis could then be applied to answer that question. In an earlier research (Dumont, Hamers, Ruyssenaars, 1977) we tried to do so. Because interpretation of factors is not suited to causal explanation, such an analysis does not explain the nature of the connexions found. That is why we did a reanalysis, formulating the problem set in terms of functional dependency. In the relevant literature a number of variables are suggested to explain arithmetic disabilities. These variables have been operationalized and are presented as independent variables to explain the variance in a group of thirteen dependent variables, which are supposed to be representative for the arithemtical performance of initial learners.
All external variables relating to environment, primary socialisation, motivation, instruction, and didactic style have been left out of consideration. We aim exclusively at variables which as abilities or aptitudes within the cognitive structure of the child are supposed to influence the initial arithmetical performance.
There are other limitations to this research apart from the choice of the ability-model as such and leaving out external variables. The sample of 120 children is a selected sample and cannot be regarded as representative for the whole Dutch population of children with arithmetic disabilities. Although conclusions will be limited for those reasons, we hope to contribute to the understanding of arithmetic problems of young children.

The problem can be formulated as follows: to what extent performances on a number of arithmetic tests are explained by a number of variables supposed to influence those performances in a causal way.

The independent variables

Selection of independent variables has been made on account of what has been put forward in the research literature as possible causes of failure in arithmetic.

- Intelligence. Common in arithmetic and intelligence should be: insight in problems and the acquisition of strategies to solve intellectual problems (Schonell and Schonell, 1965; Smith, 1964; Dumont, 1976). Especially WISC subtests as Block-Design and Insight are supposed to represent problem solving on a verbal and non-verbal level. Apart from these tests Vocabulary and Coding were included. Vocabulary was taken to demonstrate that it is indeed problem solving and that it is not just verbal intelligence in general that is involved in arithmetic. In other words: Vocabulary should not explain arithmetic disabilities. Coding was included because arithmetic can be conceptualized as the practical ability to transpose information of one code into output of another code. The flexibility required can possibly play an important role in automatizing arithmetical performances.
  Raven; Progressive Matrices were taken because this test of general intelligence could give us an indication to what extent general intelligence could be held responsible for beginnen arithmetic.
- Specific Factors. In factor analytic studies it was shown that visuo-spatial abilities are connected with mathematical performance in secondary education (Smith, 1964; Butcher, 1970; Vernon, 1971). Thus several researchers supposed that these same factors could account for arithmetic in primary education. In our study the visual factor has been operatiionalized in the variables Three-Dimensional Patters from Leiter's International Performance Scale. The test Paper Folding was taken from French's Kit of Reference Tests. The spatial ability was represented by Card Rotation from French's Kit of Reference Test and Space from a Dutch revision of Thurstone's Primary Mental Abilities Test.
- Additional Variables. Under this heading we take together some variables which have been included on the ground of several hypotheses put forward in other studies
  . Memory was operationalized as memory for words and memory for numbers. Memorizing can be regarded as a partial explanation of success or failure in arithmetic (van der Laan, 1973).

. Fluency is an ability which as automatization of the retrieval and output of verbal storage can be conceived as an indicator for automatization in learning arithmetic.
In flucency-bound, the production of words is conditioned by a task; fluency-free demands the child just to mention as many words as he can within one minute (Fruchter, 1948; Bannatyne, 1973; Hulsmans e.a., 1978).

. Classification and Seriation are two tests derived from Piaget's theory where they are conceptualized as pre-requisites and conditions for cardination and ordination (Dumont, 1966). Including these two tests, Classification from Snijders-Oomen Non-verbal Intelligence Test and Dot Estimation from Leiter's International Performance Scale, gives us the possibility to see whether these conditions still play an important role in our research sample.

The dependent variables

Arithmetical performance was assessed by

- S.R.T. (Schiedamse Reken Test). This represents a sample of all arithmetic problems which figure in traditional arithmetic instruction in all curricula, including not only computation but also arithmetic reasoning and comprehensive arithmetic, measuring and computing with money.
- Addition under 20, as an operationalization of fast and accurate performance of the first arithmetic operation.
- Subtraction under 20, analogous operationalization of the second arithmetic operation.

Addition and Subtraction were scored by assessing how many sums can be done within one minute. For S.R.T. score there is no time limit.

Sample

120 Children (98 boys and 22 girls) were selected from 12 schools for children with learning disabilities. Their age-range was between 9 and 11 with a mean of 123.8 months. Selection was based on the teacher's judgment that the child had a retardation in arithmetic of one to two years. Then this retardation was assessed by means of the S.R.T. Because we only wanted to get

children with arithmetic disabilities in our sample, a reading test was administered to sort out those children who proved to have reading problems as well as arithmetic difficulties. This idea turned out to be not very appropriate. Many children had both disabilities.

Analysis

First of all frequencies, means and standarddeviations were calculated, to compare the sample with normal primary school children.
A multiple regression analysis was carried out with 17 independent and 3 dependent variables.
On the basis of the standardised regression coefficients of the independent variables a selection was made for a second analysis with 10 independent and 3 dependent variables. Finally some independent variables were combined to trace what influence that would have on the explanation of the variance.
The sequence in which the independent variables were introduced in the analysis was determinded by the relative explanatory weight that was attributed to them on the basis of theoretical considerations.

Results

First of all means and standarddeviations are shown of all variables concerned.

Insert Table 1 about here

These means and standard deviations show the research sample to be a group of normal intelligence (Raven P.M. for 10;6 = 29; except for WISC Coding all WISC subtests lay somewhere above the mean). As was mentioned before the reading and arithmetic performances are more than one year behind the mean of their age.

The first regression analysis was carried out with the 17 independent and the 3 arithmetic tests above as dependent variables. The regression-coefficients are mentioned between parentheses. Report of significance is as usual. The independent variables were introduced in the order which is shown below,

because we expected that in that way their relative weights could be estimated in the most adequate manner.

Insert Table 2 about here

Inspection of the table of analysis shows that some variables which are supposed to play a role in explaining the variance of the dependent variables, did not come up to these expectations. From the values of the standardised regression coefficients we learn that several variables such as Raven P.M., Visualization, Classification and Seriation have no connexion with the dependent variables.
On the basis of these results a second analysis was carried out, the number of independent variables reduced to 10.

Insert Table 3 about here

At last a final reduction of the number of independent variables was established by combining the separate visuo-spatial variables and by considering them as one independent variable. The same procedure was followed with regard to free and bound fluency.
Although we notice a slight change in F-values, the overall picture of the analysis is maintained.

Insert Table 4 about here

Interpretation

Interpretation of results can be based best on table 3, where 10 selected independent variables explain the variance of the 3 dependent variables.
This selection by itself already leads to some conclusions. General Intelligence, as measured in a g-test like Raven P.M. did not sufficiently explain the different performances of these children. This proves the starting-point that arithmetic disabilities are independent of the general, overall level of intelligence, as these disabilities have to be explained by more specific factors like Block-design and Insight. Arithmetic as demanded from the child

in achievement-test like S.R.T. turns out to be primarily dependent on verbal and non-verbal problem solving. Besides we should keep in mind that age, despite its small range, is still the most powerful factor. In as far as we may conceive age as time spent on arithmetic, the conclusion would be justified that school education is effective: the more the children progress in school, the more instruction is paying off and the more their achievement improves. The tasks addition and subtraction are distinguished from general achievement and are distinguished from each other.
First of all the absence of a relationship with WISC-variables seems to point out that in addition and subtraction specific tasks are involved, which are conditioned by time-limits and can be described as automatization. Particularly addition can be explained by the content of the variable reading and fluency free. To these two tasks automatization is common. Performance of these tasks is based on output production available in an automatic way. The more the child masters automatization, the more it masters addition. Subtraction behaves in a markedly different way in all analyses. Obviously automatization is less a condition to reach good results. The connection with the subtests Coding and Memory for Numbers shows that subtraction is a different kind of information processing. The difference in frequency not only shows a much longer reaction-time and/or less accuracy of subtraction. The combination with Memory for Numbers points out to a more reflexive activity. In the process of subtraction numbers are to be memorized explicity and are to be operated on explicity. This activity contrasts very much with automatization in addition and could be termed explicitization or semi-automatization.

Conclusions

On the basis of this study with children with severe arithmetic disabilities we can conclude that generally speaking, arithmetic disabilities are more dependent on verbal and non-verbal problem solving than on general mental ability. Addition and subtraction are distinguished from general arithmetic ability as both of them are dependent on different abilities, described as automatization and explicitization.
Years of instruction proved to be the best and most powerful explanation. Specific prerequisites like classification and seriation played no role

anymore on the educational level of these pupils.

Further research should aim at task-analysis of the different tasks. The two tasks addition and subtraction although mostly considered as both sides of one operation in a logical way, prove to be two contrasting tasks in a psychological way.

Dependent variables

	M.	Sd.
1. S.R.T.	107.7	58.8
2. Addition	24.8	11.1
3. Subtraction	17.4	11.2

Independent variables

	M.	Sd.
1. Age	123.8	9.2
2. Reading	35.2	23.1
3. Raven P.M.	24.9	8.4
4. WISC Vocabulary	41.6	8.4
5. WISC Insight	16.1	4.3
6. WISC Coding	29.3	7.8
7. Paper Folding	10.1	6.3
8. WISC Block Design	24.5	10.9
9. Three Dim. Patterns	6.6	2.7
10. Card Rotation	85.8	27.9
11. Thurstone Space	14.5	4.4
12. Word Memory	8.7	3.2
13. Number Memory	4.5	1.6
14. Fluency Free	25.6	7.9
15. Fluency Bound	16.6	4.3
16. Classification	6.2	2.4
17. Seriation	7.9	3.8

Table 1.: Means and Sd by variable

	SRT		Addition		Subtraction	
1. Age	52.8**	(.24*)	0.4	(.24*)	2.9	(.09)
2. Reading	21.5**	(.31*)	19.7**	(.47*)	0.0	(.39*)
3. Raven P.M.	8.4**	(-.06)	3.3	(-.08)	0.3	(-.01)
4. WISC Vocabulary	5.5*	(-.01)	0.5	(-.14)	0.7	(.00)
5. WISC Insight	11.6**	(.18*)	0.4	(.13)	0.3	(.17)
6. WISC Coding	5.3*	(.21*)	1.5	(.07)	11.2**	(.24*)
7. Paper Folding	7.3**	(.14)	0.0	(.12)	0.0	(.12)
8. WISC Block Design	9.4**	(.19*)	0.0	(.20*)	0.1	(.16)
9. Three Dim. Patterns	0.4	(.04)	0.6	(-.00)	0.1	(.00)
10. Card Rotation	7.1**	(-.20*)	4.5*	(-.02)	0.5	(.00)
11. Thurstone Space	0.4	(.07)	5.3*	(-.13)	0.0	(-.09)
12. Word Memory	0.6	(.08)	0.4	(.03)	0.1	(.12)
13. Number Memory	1.8	(-.11)	0.0	(-.12)	7.9**	(-.27*)
14. Fluency Free	0.5	(-.00)	9.1**	(.19*)	1.3	(.08)
15. Fluency Bound	3.5	(.13)	1.8	(.17)	0.5	(.11)
16. Classification	1.1	(.07)	2.1	(-.06)	1.0	(-.07)
17. Seriation	0.7	(.06)	0.3	(.00)	1.0	(-.04)
Variance accounted for	66.1%		55.2%		51.0%	

Table 2.: Step-wise regression: F by separate variable (first analysis) in parentheses: standardised regression-coefficients.

	SRT		Addition		Subtraction	
1. Age	52.8**	(.25*)	0.4	(.21*)	2.9	(.06)
2. Reading	21.5**	(.30*)	19.7**	(.44*)	0.0	(.38*)
3. WISC Insight	16.4**	(.21*)	0.1	(.06)	0.7	(.16)
4. Paper Folding	14.2**	(.15)	1.3	(.04)	0.0	(.07)
5. WISC Block Design	11.7**	(.20*)	0.5	(.08)	0.0	(.09)
6. WISC Coding	6.5**	(.15)	1.0	(.08)	11.1**	(.24*)
7. Number Memory	0.3	(-.10)	0.6	(-.15)	6.0**	(.27*)
8. Word Memory	1.6	(.09)	0.0	(.05)	1.2	(.12)
9. Fluency Free	1.1	(0.08)	12.5**	(.17*)	0.2	(.08)
10. Fluency Bound	4.5*	(.14)	0.7	(.13)	0.1	(.10)
	63.2%		51.7%		49.9%	

Table 3.: Step-wise regression: F by separate variable (second analysis) in parentheses: standardised regression-coefficients.

	SRT	Addition	Subtraction
1. Age	52.8**	0.4	2.9
2. Reading	21.5**	19.7**	0.0
3. WISC Insight	16.4**	0.1	0.7
4. Paper Folding/5. Block Design	13.6**	0.7	0.0
6. WISC Coding	6.5**	1.0	11.1**
7. Number Memory/8. Word Memory	1.0	0.3	3.7*
9. Fluency Free/10. Fluency Bound	2.3	6.6**	0.2

Table 4.: Step-wise regression by grouped variable

Literature

Bannatyne, A. — Language, Reading and Learning Disabilities. Illinois, 1973.

Borghouts-van Erp, J.W.M. — Rekenstoornissen. Een pleidooi voor visie. Tijdschrift voor Orthopedagogiek, juni 1978, 6.

Brus, B.Th. en Voeten, M.J.M. — Eén-minuut-test. Verantwoording en handleiding. Nijmegen, 1973.

Butcher, H.J. — Human Intelligence, its Nature and Assessment. London, 1970.

Coan, R.W. — Facts, Factors and Artifacts: The quest for psychological meaning. Psychological Review, 1964, Vol. 71, no: 2, 123 - 140.

Cronbach, L.J. — Essentials of Psychological Testing. Part II, 197 - 451, London, 1970.

Dodwell, P.C — Relationships between the understanding of the logic of classes and of cardinal number in children. Canadian Journal of Psychology, 1962, 14, 191 - 205.

Dumont, J.J. — De ontwikkeling van de intelligentie. 's-Hertogenbosch, 1966. pp. 251 - 263.

Dumont, J.J. — Leerstoornissen I. Theorieën en Modellen. Rotterdam, 1976.

Dumont, J.J. en Kok, J.F. Curriculum Schoolrijpheid. Deel I. 's-Hertogenbosch, 1970.

Dumont, J.J.; Hamers, J.H.M. en Ruyssenaars, A.J.J.M. Rekenstoornissen. De samenhang van technisch en begrijpend rekenen met enkele psychologische vaardigheden. Pedagogische Studiën, 1977 (54), 386 - 397.

Eysenck, H.J. Intelligence Assessment: a theoretical and experimental approach. British Journal of Educational Psychology, 1967, 37, pp. 81 - 98.

Ferguson, G.A. Human Abilities. Annual Review of Psychology, 1965, 16, 39 - 62.

French, J.W.; Ekstrom, R.B. en Price, L.A. Kit of Reference Tests for Cognitive Factors. New Yersey, 1963.

Fruchter, B. The nature of verbal fluency. Educational and Psychological Measurement, 1948, 8, 33 - 47.

Fruchter, B. Measurement of Spatial Abilities. Educational and Psychological Measurement, 1954, 14, 2, 387 - 395.

Geffen van, L.M.H.J. Wat is Intelligentie? De struktuurbenadering. Gedrag, tijdschrift voor Psychologie, 19, 6, 1971.

Hamel, B.R. Children from 5 to 7. Rotterdam, 1974.

Heesen, H.; Strelitski D. en Wissel van der, A.
Schiedamse Rekentest. Handleiding.
Groningen, 1970.

Hulsmans, Cl.

Humphreys, L.G.
The organisation of human abilities.
American Psychologist, 1962, 17, 475 - 483.

Kaufmann, A.S.
Piaget en Gesell: A psychometric analysis of tests built from their tasks.
Children Development, 1971, 42, 1341 - 1360.

Kema, G.N.
P.M.A. 2 - 4.
Nederlandse bewerking.
Amsterdam. 1972.

Koster, K.B.
De ontwikkeling van het getalbegrip op de kleuterschool.
Groningen, 1975.

Laan, H. van der
Leren lezen, schrijven en rekenen.
Groningen, 1973.

Michael, W.B.
A suggested research approach to the identification of psychological processes associated with spatial-visualisation factors.
Educational and Psychological Measurement, 1954, 14, 2, 401 - 406.

Nunnally, J.C.
Psychometric Theory.
New York, 1967.

Piaget, J.; Resag, K.; Fricke, A.; Hiele, P.M. van en Odenbach, K.
Rekenonderwijs en getalbegrip.
Baarn, 1973.

Rapaport, D.; Gill, M.M. en Schafer, R.
Diagnostic Psychological Testing.
New York, 1968, ch. II, pp. 64 - 68.

Raven, J.C.
Manual for the Standard Progressive Matrices.
London, 1965.

Russell, G. en Leiter, Ph.D.
Examiners Manual for the Leiter International Performance Scale.
Chicago, Illinois, 1969.

Schonell, F.J. en Schonell, F.E.
Diagnosis and remedial teaching in artihmetic.
London, 1965.

Smith, J.M.
Spatial Ability. Its educational and social significance.
San Diego, 1964.

Snijders, J.Th. en Snijders-Oomen, N.
Nicht-Verbale Intelligenzuntersuchung.
Groningen, 1967.

Thurstone, T.G.
Primary Mental Abilities of Children,
Educational and Psychological Measurement,
1941, 1, 105 - 116.

Vernon, P.E.
The Structure of Human Abilities.
London, 1971.

Wechsler, D.
Wechsler Intelligence Scale for Children.
Manual.
New York, 1949.

Wouters, P.A.M. en Boon van Ostade, A.H.
Intelligentie: een algemene faktor of niet?
Gedrag, tijdschrift voor Psychologie,
1973, 1, 341 - 350.